2019 Year 11 Mathematics Specialist Semester One Exam Preparation

Question 1 **(9 marks)**

(a) Evaluate ${}^{25}P_{19} \div {}^{23}P_{20}$ (3 marks)

$$\frac{25!}{6!} \div \frac{23!}{3!} = \frac{25!}{6!} \times \frac{3!}{23!}$$

$$= \frac{25 \times \cancel{24} \times \cancel{23!} \times \cancel{3!}}{\cancel{6} \times 5 \times \cancel{4} \times \cancel{3!} \times \cancel{23!}}$$

$$= 5$$

✓ Express using factorials

✓ Eliminate factorials

✓ Evaluates

(b) Express 8! + 7! + 6! in the form $a^2b!$, where a and b are positive integers. (3 marks)

$$8! + 7! + 6! = 8 \times 7 \times 6! + 7 \times 6! + 6!$$

$$= (8 \times 7 + 7 + 1) \times 6!$$

$$= 64 \times 6!$$

$$= 8^2 \times 6!$$

✓ Factors out lowest factorial

✓ Simplifies

✓ Writes in required form

(c) Show that for $n \in Z, n \geq 1$, the sum $(n+2)! + (n+1)! + n!$ can always be expressed in the form $a^2b!$ where a and b are positive integers. (3 marks)

$$(n+2)! + (n+1)! + n!$$

$$= (n+2)(n+1)n! + (n+1)n! + n!$$

$$= \left((n+2)(n+1) + (n+1) + 1\right)n!$$

$$= (n^2 + 3n + 2 + n + 1 + 1)n!$$

$$= (n^2 + 4n + 4)n!$$

$$= (n+2)^2 n!$$

where $a = n+2$ & $b = n$

✓ Factors out n! as a factor

✓ Clearly shows composition of (n^2+4n+4) as the second factor

✓ Simplifies and writes in required form

Question 2 **(10 marks)**

(a) If $\mathbf{a} = 3\mathbf{i} - 5\mathbf{j}$, $\mathbf{b} = -2\mathbf{i} + 7\mathbf{j}$, Determine

(i) $\boldsymbol{2a - 3b}$ (2 marks)

$2\vec{a} = 6\vec{i} - 10\vec{j}$, $3\vec{b} = -6\vec{i} + 21\vec{j}$

$\therefore\ 2\vec{a} - 3\vec{b} = 6\vec{i} - 10\vec{j} + 6\vec{i} - 21\vec{j}$

$= 12\vec{i} - 31\vec{j}$

✓ Determines scalar multiples

✓ Determines difference

(ii) $|\boldsymbol{a} + \boldsymbol{b}|$ (2 marks)

$\vec{a} + \vec{b} = 3\vec{i} - 5\vec{j} + (-2\vec{i} + 7\vec{j})$

$= \vec{i} + 2\vec{j}$

$|\vec{a} + \vec{b}| = \sqrt{1^2 + 2^2} = \sqrt{5}$

✓ Determines sum

✓ States exact value

(iii) The unit vector **c** that is parallel and in the same direction as $\boldsymbol{b} - \boldsymbol{a}$ (3 marks)

$\vec{c} = \vec{b} - \vec{a} = -2\vec{i} + 7\vec{j} - (3\vec{i} - 5\vec{j})$

$= -5\vec{i} + 12\vec{j}$

$|\vec{c}| = \sqrt{(-5)^2 + 12^2} = 13$

$\therefore\ \hat{c} = -\frac{5}{13}\vec{i} + \frac{12}{13}\vec{j}$

✓ Determines $\vec{b} - \vec{a}$

✓ Determines magnitude of $\vec{b} - \vec{a}$

✓ States unit vector

(b) Given that **d** and **e** are non-parallel vectors, find the values of γ and μ in the following expression: $(\gamma + \mu - 4)\boldsymbol{d} = (\mu - 3\gamma)\boldsymbol{e}$ (3marks)

$$\begin{cases} \gamma + \mu - 4 = 0 \\ \mu - 3\gamma = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \gamma = 1 \\ \mu = 3 \end{cases}$$

✓ Scalars equal to zero

✓ Determines γ

✓ Determines μ

Question 3 **(7 marks)**

(a) A body moves from $P(2,-3)$ to $Q(-2,1)$.

(i) Determine the displacement vector $\overrightarrow{PQ}$ in component form. (1 mark)

$$\overrightarrow{PQ} = \begin{pmatrix} -2-2 \\ 1-(-3) \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \begin{matrix} \rightarrow i \\ \rightarrow j \end{matrix}$$

$$\overrightarrow{PQ} = -4\vec{i} + 4\vec{j}$$

✓ Express in component form

(ii) Determine the magnitude of the vector $\overrightarrow{PQ}$. (1 mark)

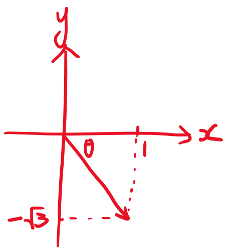
$$|\overrightarrow{PQ}| = \sqrt{(-4)^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$$

✓ States magnitude

(b) A force of $6\mathbf{i} - 6\sqrt{3}\mathbf{j}$ N acts on a body. Determine the magnitude of the force and the angle its direction makes with the positive x-axis. (2 marks)

$$|\vec{F}| = \sqrt{6^2 + (6\sqrt{3})^2} = 12\,N$$

$$\vec{F} = 6(\vec{i} - \sqrt{3}\vec{j})$$



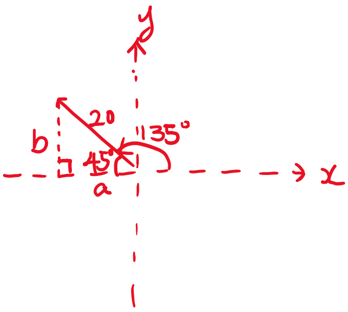
$$\tan\theta = -\sqrt{3}$$

$$\theta = -60^\circ$$

✓ States magnitude

✓ States angle

(c) A body moves with a velocity of 20 ms^{-1} at an angle of 135° with the positive x-axis. Express the velocity of the body in the form $a\mathbf{i} + b\mathbf{j}$, where a and b are constants. (3 marks)



$$a = 20 \times \cos 135^\circ = 20 \times \left(-\frac{\sqrt{2}}{2}\right) = -10\sqrt{2}$$

$$b = 20 \times \sin 135^\circ = 20 \times \frac{\sqrt{2}}{2} = 10\sqrt{2}$$

$$\therefore \vec{v} = -10\sqrt{2}\,\vec{i} + 10\sqrt{2}\,\vec{j} \text{ m/s}$$

✓ Determines expressions for a and b

✓ Simplifies a and b

✓ States in required form

Question 4 **(8 marks)**

(a) Write the inverse of the following true statement and comment on the truth of the inverse statement. "If the discriminant of the quadratic formula is zero, then the quadratic has just one real root." (2 marks)

Inverse : "If the discriminant of the quadratic formula is not zero, then the quadratic does not have just one real root."

True.

✓ Changes 'if P then Q' to 'if not P then not Q'.

✓ Indicates statement is true.

(b) Write the converse of the following true statement and comment on the truth of the converse statement. "If $x > 3$ then $x > 2$". (2 marks)

Converse : "If $x > 2$ then $x > 3$."

False.

✓ Changes 'if P then Q' to 'if Q then P'.

✓ Indicates statement is false.

(c) Determine the truth of the following statements, using an example or counter-example to support each answer.

(i) If $z \in \mathbb{R}$ and z^3 is an even number then z is an even number. (2 marks)

Statement is false.

If $z^3 = 6$ (even) then $z = \sqrt[3]{6}$ (not even, irrational)

✓ States false

✓ Supplies counter-example

(ii) $\forall\, x > 1$ *and* $x \in$ (2 marks)

Statement is false.

If $x = 5$, then $x^2 - x + 1 = 25 - 5 + 1 = 21$,

but 21 is not a prime.

✓ States false.

✓ Supplies counter-example using integers.

Question 5 **(7 marks)**

(a) Write down and prove the contrapositive of the statement "if $n^2 + 2n + 6$ is odd, then n is odd". (3 marks)

Contrapositive : "If n is even, then n^2+2n+6 is even."

Proof: Let $n = 2a$, $a \in \mathbb{Z}$

$\therefore \quad n^2+2n+6 = (2a)^2 + 2\times(2a) + 6$

$= 4a^2 + 4a + 6$

$= 2(2a^2 + 2a + 3)$

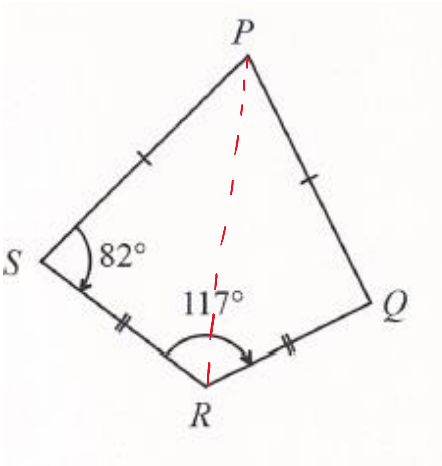
$\therefore$ Since contrapositive statement is true, then original statement is true.

✓ Correct contrapositive statement.

✓ Correct proof of even.

✓ Indicates original statement is true.

(b) Prove by contradiction that it is impossible to draw a circle through the vertices of quadrilateral shown below. (4marks)



- Assume that quadrilateral PQRS is cyclic.

  Then $\angle S + \angle Q = 180°$

- Since $PS = PQ$, $SR = QR$, $PR = PR$,

  $\triangle PSR \cong \triangle PQR$ (SSS).

  Then $\angle S = \angle Q = 82°$. $\angle S + \angle Q = 164°$

- This contradicts the assumption that the quatrilateral is cyclic, and hence it is not.

- Hence the original statement "It is impossible to draw a cicle through the vertices shown" is true.

✓ Correct assumption

✓ States congruent triangles

✓ States specific contradiction

✓ Indicates original statement is true